Efficient Simulation of Electric Line Parameters for Cables

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Introduction

One important aspect of the design of electric power systems is electromagnetic transient (EMT) analysis of cables. Numerical methods such as the Finite Element method enable direct detailed modeling of field results for specific transient cases, but this is impractical for many purposes. As an example, consider the results in Figure 1.

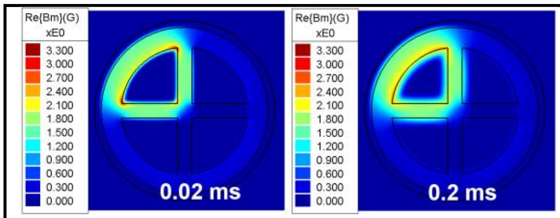

Figure 1: B field at 0.02 and 0.2 ms after turning on DC current

Similarly, one can display directions of fields, power density, current distributions, etc. in many plot formats. One can also query values at specific points and produce tables and graphs of the time dependence, as in Figure 2.

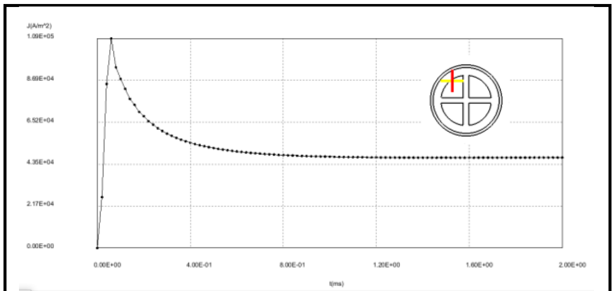

Figure 2: Current density at a point versus time

All of these analyses have many uses, but are generally not useful for the purpose of studying the cable as component in a system or circuit. Instead, for those purposes it is normal to conduct a frequency dependent analysis of self- and mutual-impedances, and the constituent parameters (capacitance, inductance, resistance, conductance) for the conductors in a system. Those results can be used very efficiently as needed for a multitude of specific transient cases within some circuit using dedicated software.

Before using the results, in general one needs to verify their quality. That means assessing the accuracy of a large number of individual impedance results relative to one's needs. Furthermore, each set of results may take significant time to calculate.

This paper is based on some of the research done to determine good default solver setup for CABLES – a new program created from parts of ELECTRO and OERSTED. The goal was to provide a minimalistic interface dedicated to solving electrical parameters in sector cables. This paper is a case study for efficiently assessing the accuracy of the results for a specific four sector cable and sheath, and adjusting the solver conditions as necessary. The discussion focusses on how to effectively use options commonly available in FEM and related software to determine efficient and trusted protocols for analyzing models of a given type. The model and results to compare with come from publications by Shafieipour et al.[1, 2].

Test Model Description

The test model consists of a 4 sector cable surrounded by a sheath of inner radius 25 mm and outer radius 27 mm. The 4 sectors can be understood by geometric subtraction of a "+" shape taken out of a circle.

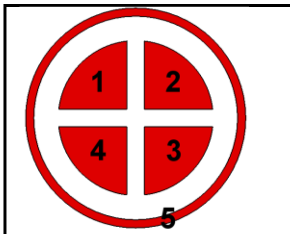

Figure 3: Test Model Conductor Numbers

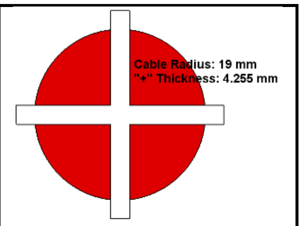

Figure 4: Sector Construction

All conductors are assigned as copper. For capacitance calculations this could be understood as a 4 conductor model, since the sectors will not influence anything beyond the sheath. However, for the purpose of the total impedance one must account for the skin effect, hence the fields penetrating beyond the sheath. Thus the total problem is considered to be a 5 conductor impedance matrix with a 1 m ground/return path.

$$[Z] = \begin{bmatrix} Z_{1,1} & Z_{1,2} & Z_{1,3} & Z_{1,4} & Z_{1,5} \\ Z_{2,1} & Z_{2,2} & Z_{2,3} & Z_{2,4} & Z_{2,5} \\ Z_{3,1} & Z_{3,2} & Z_{3,3} & Z_{3,4} & Z_{3,5} \\ Z_{4,1} & Z_{4,2} & Z_{4,3} & Z_{4,4} & Z_{4,5} \\ Z_{5,1} & Z_{5,2} & Z_{5,3} & Z_{5,4} & Z_{5,5} \end{bmatrix}$$

However, one can exploit the symmetry of the model to note that (for example): the self-effects of the first 4 conductors are all the same; that the interaction between 1 and 2 is the same as between 1 and 3; etc. Hence for this case the testing is reduced to the following 5 impedances.

$$Z_1 \stackrel{\text{def}}{=} Z_{1,1} = Z_{2,2} = Z_{3,3} = Z_{4,4}$$

$$Z_2 \stackrel{\text{def}}{=} Z_{1,2} = Z_{1,4} = Z_{2,1} = Z_{2,3} = Z_{3,2} = Z_{3,4} = Z_{4,1} = Z_{4,3}$$

$$Z_3 \stackrel{\text{def}}{=} Z_{1,3} = Z_{2,4} = Z_{3,1} = Z_{2,4}$$

$$Z_4 \stackrel{\text{def}}{=} Z_{1,5} = Z_{2,5} = Z_{3,5} = Z_{4,5} = Z_{5,1} = Z_{5,2} = Z_{5,3} = Z_{5,4}$$

$$Z_5 \stackrel{\text{def}}{=} Z_{5,5}$$

Numerical Methods to be Examined

Figure 5 depicts how the geometry in discretized for analysis by two common methods:

1) FEM (Finite Element Method) is the most widely known and used method due to its simplicity to implement. The model geometry and immediately surrounding space are filled with a mesh used to compute the spatial distribution of some potential, such as voltage or the vector magnetic potential. Specific results are obtained through appropriate relationships to that potential (e.g. electric field = -grad(voltage).
2) BEM (Boundary Element Method, aka MoM, Method of Moments). A surface mesh is used to compute the distribution of appropriate sources, such as the charge distribution in an electric problem. Specific results are obtained through relationships to the source (e.g. electric field at a point is the sum of all charge divided by the square of the distance to each charge).

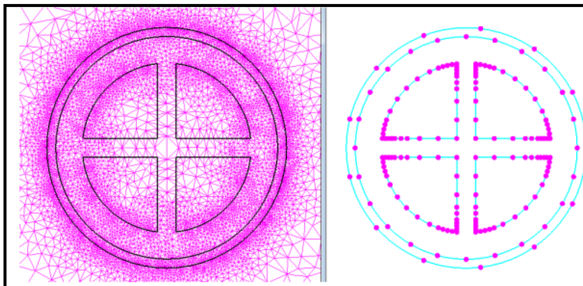

Figure 5: Discretization for two solver methods

Shafieipour et al. use the commercial software COMSOL to provide reference FEM results and compare these results with their own MoM implementation. This paper will use the commercial software OERSTED[3] which contains both methods. Figure 6 shows their Z1 results beside typical OERSTED results for frequencies from 1 Hz to 1 MHz.

Method	Z_1
FEM, 1Hz	8.08E-05+6.31E-06i
Proposed, 1Hz	8.21E-05+6.19E-06i
FEM, 10Hz	8.15E-05+6.30E-05i
Proposed, 10Hz	8.20E-05+6.31E-05i
FEM, 60Hz	9.69E-05+3.68E-04i
Proposed, 60Hz	9.71E-05+3.68E-04i
FEM, 100Hz	1.11E-04+6.01E-04i
Proposed, 100Hz	1.11E-04+6.01E-04i
FEM, 1kHz	2.82E-04+5.58E-03i
Proposed, 1kHz	2.40E-04+5.52E-03i
FEM, 10kHz	1.39E-03+5.34E-02i
Proposed, 10kHz	9.91E-04+5.34E-02i
FEM, 100kHz	2.45E-03+5.24E-01i
Proposed, 100kHz	2.76E-03+5.28E-01i
FEM, 1MHz	2.50E-03+5.24E+00i
Proposed, 1MHz	8.46E-03+5.26E+00i

Shafieipour Results

	Z_Re	Z_Im
FEM	8.13E-05	6.31E-06
BEM	8.16E-05	6.46E-06
FEM	8.20E-05	6.31E-05
BEM	8.25E-05	6.36E-05
FEM	9.75E-05	3.68E-04
BEM	9.84E-05	3.70E-04
FEM	1.12E-04	6.01E-04
BEM	1.13E-04	6.04E-04
FEM	2.81E-04	5.58E-03
BEM	2.80E-04	5.61E-03
FEM	1.17E-03	5.36E-02
BEM	1.10E-03	5.40E-02
FEM	3.57E-03	5.28E-01
BEM	3.42E-03	5.31E-01
FEM	1.12E-02	5.26E+00
BEM	1.09E-02	5.27E+00

Oersted Results

Figure 6: Comparison of multiple Z_1 calculations

The results are all similar at low frequencies, but at high frequency the resistance (real part of Z) is a small fraction of the impedance and thus will require more stringent solver settings than the low frequency cases to get accurate results. In particular, their MoM resistance results at 1 MHz are close to the OERSTED results, but quite discrepant from their COMSOL (FEM) results. Determining how to get better agreement at high frequency was not a focus of their work, but in the following section we will discuss the value of working with more than one solver and of reconciling differences as needed.

Note that the preceding OERSTED calculation does not include the capacitance. The close agreement in imaginary results suggests that inductance is the main contribution to the imaginary part of Z for this cable.

FEM / BEM Comparison

As mentioned previously, FEM is the most common numerical method not because it is usually the best, but because it is the easiest for a software developer to produce. At the outset of new work, when developing protocols for solving models of a given type, it is worth running a comparison of multiple solver types (as available), especially a spatial mesh (differential calculus) type (FEM, FDTD) compared to a geometry mesh (integral calculus) type (BEM, FMM). In this comparison the calculations are so distinct that one can consider them as independent verification. The difference between results is then a good indicator of the calculation error. Refining each solution until they agree within some required error margin will identify both which solver is better for the given model type, and also identify efficient solver settings. Table 1 shows various cases tested using increasing numbers of FEM and BEM self-adaptive solution steps. In the first step the model is solved with a default mesh. The error on each mesh element is estimated, then on the second step the elements with the highest error are subdivided. The procedure continues for all subsequent steps. The time taken is not linear. Not only is the number of elements increased fractionally (typically 30%) on each step, but the number of calculations performed is not proportional to the number of elements. More commonly it is proportional to the square or cube of the number of elements.

Method	# Steps	Time (s)	R (Ohms)	Discrepancy
BEM	1	2	0.0107631	-2.242%
BEM	2	5	0.0109343	-0.687%
BEM	3	9	0.0110074	-0.023%
BEM	4	15	0.0110157	0.052%
BEM	5	26	0.0110153	0.048%
BEM	6	44	0.0110142	0.039%
BEM	7	79	0.0110134	0.032%
BEM	8	148	0.0110128	0.026%
BEM	9	281	0.0110116	0.015%
BEM	10	540	0.0110108	0.008%
BEM	11	1070	0.0110104	0.004%
BEM	12	2197	0.0110099	0.000%
FEM	1	26	0.0117398	6.629%
FEM	2	60	0.0111629	1.389%
FEM	3	212	0.0111289	1.081%
FEM	4	714	0.0111250	1.045%

Table 1: FEM and BEM Results for R at 1 MHz

The entire Z matrix was computed in the times shown above. Since resistance at 1 MHz was previously the most difficult calculation, it was selected for as the standard result to determine the best solver conditions. Both solvers show a reasonably monotonic convergence toward a consistent answer as the number of steps increases. Convergence is one of the common assessment criteria for a solver. The resistance for 12 BEM self-adaptive steps was selected for the standard reference.

Suppose the goal is to produce numbers accurate within 1%. The results in Table 1 suggest that one would achieve this with 2 adaptive steps, taking 5 seconds using the BEM solver, but would require a few minutes or more (depending on how close to 1% was acceptable) to achieve with the FEM solver. Hence, for the OERSTED tool the choice of BEM to solve with 2 adaptive steps is clear.

In fact, the two solvers appear to be converging to answers approximately 1% different. That suggests some subtle systematic difference in how each method is solving, even though they are using the same model. If better than 1% is required, it would be recommended to track down the source of that discrepancy.

Wider Study of Results for a Fixed Number of Self-Adaptive Steps

Having determined that the results were in close general agreement with published results based on two independent codes, and that the BEM solver was clearly superior for speed of achieving a desired accuracy, a wider study was run on the time taken to solve for various number of BEM self-adaptive steps. In figures 7 and 8 below the solution time is the time taken to solve all 7 frequencies and compute the entire Z matrix. The discrepancy (error) is the relative difference between the value obtained with the given number of steps, and the value obtained with 12 adaptive steps. The left-most point on each curve is one step, the next point is two steps, etc.

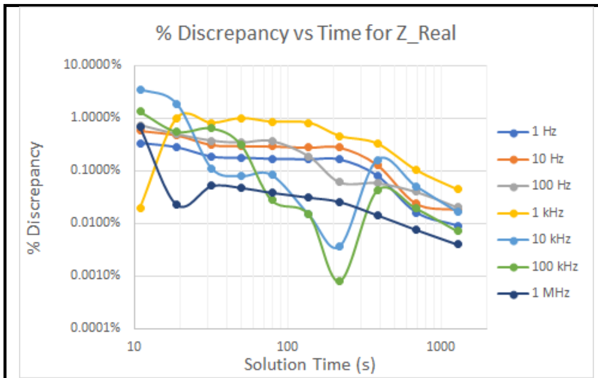

Figure 7: Real part error versus total solution time

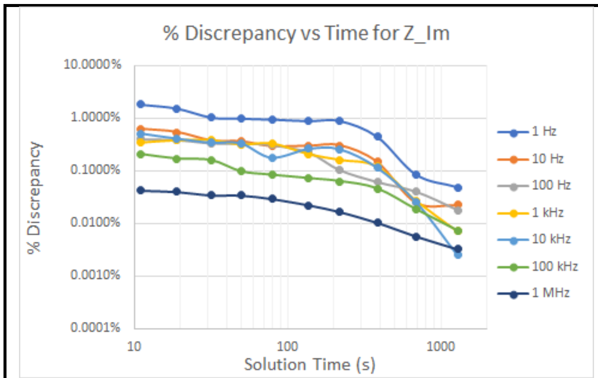

Figure 8: Imag. part error versus total solution time

All curves show a convergence towards a fixed value as more adaptive steps are taken. Most curves would confirm the previous result that for 1% accuracy it is sufficient to do 2 or 3 adaptive steps. However, each figure shows one curve that stays near the 1% level for several steps, pushing the total solution time to several minutes to ensure better than 1% for all frequencies in this range.

Examining the details for the solving process it is also seen that the higher frequencies take substantially longer to solve than the lower frequencies. This is because once skin effects become comparable to the mesh size the solver creates more elements.

Frequency	Time(s)
1 Hz	14
10 Hz	17
100 Hz	19
1 kHz	14
10 kHz	15
100 kHz	64
1 MHz	545

Table 2: Solution time for each frequency

With each frequency potentially taking a significantly different time for the same number of steps, and with the number of steps required to achieve a given error varying substantially between the different frequencies, it is worth changing from a fixed number of steps to using an exit criteria that selects the number of steps as the solving proceeds.

Results for Self-Adaptive Steps with an Exit Criteria Given

Rather than running a fixed number of steps, an exit criteria is introduced so the solver can assess whether to stop after each adaptive step. This criteria will perform some global analysis on the solution, such as the average over all boundaries of the discrepancy between a computed boundary condition and the known value. If the solution has achieved the exit criteria

These results are shown in figures 9 and 10. Note that when the exit criteria causes the solver to perform 12 adaptive steps, the discrepancy computes as zero so no point appears on the curve.

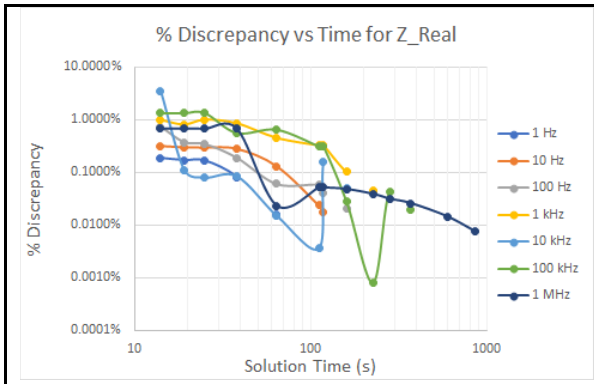

Figure 9: Real part error versus total solution time

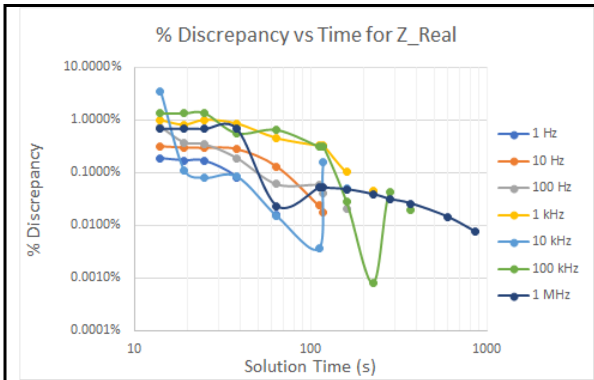

Figure 10: Imag. part error versus total solution time

In the results above, all curves are below 1% with a total solution time of less than a minute. This optimum case for beating 1% is an exit criteria of 0.004 and took 38 seconds.

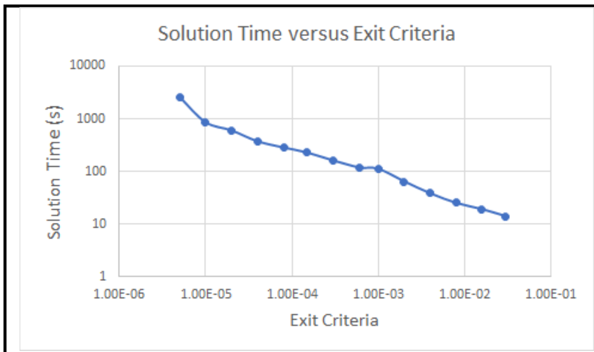

Figure 11: Total Solution time Versus Exit Criteria

Final Proposal for CABLES

One last observation is that about half the solution time for a given number of steps is taken on the previous steps. The solution time curves are plotted logarithmically because each step takes approximately twice as long as the previous step. Suppose one had a scenario where the first step took 1 second, the second step took 2 seconds, the third step took 4 seconds, the fourth step took 8 seconds, etc. In such a sequence approximately half the solution time is the final step and half is all the previous steps. So, is it possible to know in advanced a sufficient final mesh and thus use a fixed mesh rather than self-adapting? If so, one could potentially cut the solution time approximately in half.

Clearly one must do at least one adaptive solution, but one could potential use its mesh for all other frequencies. Since 1 MHz is the most demanding frequency, it was selected. The results below show the error calculated when self-adapting to 0.004 for all frequencies on the left, and on the right it shows the errors when only 1 MHz is self-adapted, then its mesh is fixed and used for all other frequencies.

Frequency	Self Adapt Exit Criteria 0.4%	Manual Elements from 1 MHz
	Z_Real	
1 Hz	0.08%	0.02%
10 Hz	0.28%	0.03%
100 Hz	0.19%	0.04%
1 kHz	0.85%	0.04%
10 kHz	0.08%	0.05%
100 kHz	0.56%	0.10%
1 MHz	0.69%	0.69%
	Z_Imaginary	
1 Hz	0.45%	0.09%
10 Hz	0.30%	0.03%
100 Hz	0.22%	0.02%
1 kHz	0.34%	0.02%
10 kHz	0.18%	0.00%
100 kHz	0.17%	0.06%
1 MHz	0.04%	0.04%

Table 3: Comparing Self-Adapt with a hybrid self-adapt them manual algorithm

The hybrid approach produces superior accuracy for all frequencies below 1 MHz as compared to self-adapting at each frequency. However, the time taken was the motivation for this method. That also worked out as hoped.

The results on the left took a total of 38 seconds. For the results on the right, solving to 0.004 at 1 MHz took 5 seconds, then solving all other frequencies took 13 seconds. Thus, the total time for the hybrid algorithm was approximately 18 seconds. Thus, the algorithm performed a little better than halving the calculation time.

Noting that the adaptive step took a significant fraction of the total time for the hybrid algorithm, and that this analysis is for a small sampling of frequencies over the range of interest, when producing a large set of results for a system simulation the hybrid algorithm may perform faster still relative to adapting every step because the relatively slow step will now one out of a much larger number.

Summary

This paper has presented benchmark and timing data used to optimize some aspects of how the OERSTED solver will be utilized in the new CABLES program. Every solver has its own distinctive aspects for how to adapt a mesh, interpret an "exit criteria" number, etc. Hence, the specific details and conclusions are only directly applicable to the OERSTED solver itself. However, the reasoning and general approaches could be adapted to other numeric solvers when searching for default settings or protocols for a new project or model type.

References

[1] "Fast Computation of the Electrical Parameters of Sector-Shaped Cables using Single-Source Integral Equation and 2D Moment-Method Discretization"
M. Shafieipour, J. De Silva, A. Kariyawasam, A. Menshov, V. Okhmatovski. *International Conference on Power Systems Transients (IPST2017)*; 2017

[2] "Efficiently computing the electrical parameters of cables with arbitrary cross-sections using the method-of-moments"
M. Shafieipour, Z. Chen, A. Menshov, J. De Silva, V. Okhmatovski. *Electric Power Systems Research* **162**, p. 37-49; 2018

[3] OERSTED. Commercially available FEM and BEM software for 2D eddy current analysis. Available from Integrated Engineering Software.

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